Chapter 22

Solutions

2 a. Estimate of population total $= N\bar{x} = 400(75) = 30\,000$

b. Estimate of standard error $= Ns_{\bar{x}}$

$$Ns_{\bar{x}} = 400\left(\frac{8}{\sqrt{80}}\right)\sqrt{\frac{400-80}{400}} = 320$$

c. $30\,000 \pm 2(320)$ or 29 360 to 30 640

4 73

6 a. Assume $(N - n)/N \approx 1$

$p = 0.55$

$$s_p = \sqrt{\frac{(0.55)(0.45)}{504}} = 0.0222$$

b. $p = 0.31$

$$s_p = \sqrt{\frac{(0.31)(0.69)}{504}} = 0.0206$$

c. The estimate of the standard error in part (a) is larger because p is closer to 0.50.

d. Approximate 95% confidence interval: $0.55 \pm 2(0.0222)$ or 0.506 to 0.594

e. Approximate 95% confidence interval: $0.31 \pm 2(0.0206)$ or 0.269 to 0.351

8 a. Stratum 1: $\bar{x}_1 = 138$
Stratum 2: $\bar{x}_2 = 103$
Stratum 3: $\bar{x}_3 = 210$

b. Stratum 1, $\bar{x}_1 = 138$

$$s_{\bar{x}_1} = \left(\frac{30}{\sqrt{20}}\right)\sqrt{\frac{200-20}{200}} = 6.3640$$

Approximate 95% confidence interval is:
$138 \pm 2(6.3640)$ or 125.3 to 150.7

Stratum 2, $\bar{x}_2 = 103$

$$s_{\bar{x}_2} = \left(\frac{25}{\sqrt{30}}\right)\sqrt{\frac{250-30}{250}} = 4.2817$$

Approximate 95% confidence interval is:
$103 \pm 2(4.2817)$ or 94.4 to 111.6

Stratum 3, $\bar{x}_3 = 210$

$$s_{\bar{x}_3} = \left(\frac{50}{\sqrt{25}}\right)\sqrt{\frac{100-25}{100}} = 8.6603$$

Approximate 95% confidence interval is:
$210 \pm 2(8.6603)$ or 192.7 to 227.3

c. $$\bar{x}_{st} = \left(\frac{200}{550}\right)138 + \left(\frac{250}{550}\right)103 + \left(\frac{100}{550}\right)210$$
$$= 50.1818 + 46.8182 + 38.1818$$
$$= 135.18$$

$$s_{\bar{x}_{st}} = \sqrt{\left(\frac{1}{(550)^2}\right)\left(200(180)\frac{(30)^2}{20} + 250(220)\frac{(25)^2}{30} + 100(75)\frac{(50)^2}{25}\right)}$$
$$= \sqrt{\left(\frac{1}{(550)^2}\right)3515833.3} = 3.4092$$

Approximate 95% confidence interval is:
$135.1818 \pm 2(3.4092)$ or 128.4 to 142.0

10 a. Stratum 1: 0.288 to 0.712
Stratum 2: 0.638 to 0.928
Stratum 3: 0.069 to 0.351

b. $p_{st} = 0.5745$

c. 0.0519

d. 0.471 to 0.678

12 a. $n = 28$, $n_1 = 10$, $n_2 = 14$, $n_3 = 4$

b. $n = 33$, $n_1 = 10$, $n_2 = 13$, $n_3 = 9$

14 a. $$\bar{x}_c = \frac{\Sigma t_i}{\Sigma M_i} = \frac{750}{50} = 15$$

$$\hat{\tau} = M\bar{x}_c = 300(15) = 4500$$

$$p_c = \frac{\Sigma a_i}{\Sigma M_i} = \frac{15}{50} = 0.30$$

b. $$\Sigma(t_i - \bar{x}_c M_i)^2 = [95 - 15(7)]^2 + [325 - 15(18)]^2 + [190 - 15(15)]^2 + [140 - 15(10)]^2$$
$$= (-10)^2 + (55)^2 + (-35)^2 + (-10)^2$$
$$= 4450$$

$$s_{\bar{x}_c} = \sqrt{\left(\frac{25-4}{(25)(4)(12)^2}\right)\left(\frac{4450}{3}\right)} = 1.4708$$

$$s_{\hat{\tau}} = Ms_{\bar{x}_c} = 300(1.4708) = 441.24$$

$$\Sigma(a_i - p_c M_i)^2 = [1 - 0.3(7)]^2 + [6 - 0.3(18)]^2 + [6 - 0.3(15)]^2 + [2 - 0.3(10)]^2$$
$$= (-1.1)^2 + (0.6)^2 + (1.5)^2 + (-1)^2$$
$$= 4.82$$

$$s_{p_c} = \sqrt{\left(\frac{25-4}{(25)(4)(12)^2}\right)\left(\frac{4.82}{3}\right)} = 0.0484$$

c. Approximate 95% confidence interval for population mean: $15 \pm 2(1.4708)$ or 12.1 to 17.9

d. Approximate 95% confidence interval for population total: $4500 \pm 2(441.24)$ or 3618 to 5382

e. Approximate 95% confidence interval for population proportion: $0.30 \pm 2(0.0484)$ or 0.203 to 0.397

16 a. Estimate of mean age of mechanical engineers: 40 years

b. Estimate of proportion attending local university: 0.70

c. 35.9 to 44.1

d. 0.523 to 0.877